

# Mathematics (MATH)

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## Mathematics (MATH) Courses

### MATH 5001 [0.5 credit] (MAT 5144)

#### Commutative Algebra

Prime spectrum of a commutative ring (as a topological space); localization of rings and modules; tensor product of modules and algebras; Hilbert's Nullstellensatz and consequences for finitely generated algebras; Krull dimension of a ring; integral dependence, going-up, going-down; Noether Normalization Lemma and dimension theory.

### MATH 5002 [0.5 credit] (MAT 5149)

#### Algebraic Geometry

Brief overview of commutative algebra, Hilbert's Nullstellensatz, algebraic sets, and Zariski topology. Affine and projective varieties over algebraically closed fields. Regular functions and rational maps. Additional topics.

### MATH 5003 [0.5 credit] (MAT 5122)

#### Banach Algebras

Commutative Banach algebras; the space of maximal ideals; representation of Banach algebras as function algebras and as operator algebras; the spectrum of an element. Special types of Banach algebras: for example, regular algebras with involution, applications.

### MATH 5005 [0.5 credit] (MAT 5127)

#### Complex Analysis

Complex differentiation and integration, harmonic functions, maximum modulus principle, Runge's theorem, conformal mapping, entire and meromorphic functions, analytic continuation.

### MATH 5007 [0.5 credit] (MAT 5125)

#### Real Analysis I (Measure Theory and Integration)

General measure and integral, Lebesgue measure and integration on  $\mathbb{R}$ , Fubini's theorem, Lebesgue-Radon-Nikodym theorem, absolute continuity and differentiation, LP-spaces. Selected topics such as Daniell-Stone theory. Also offered at the undergraduate level, with different requirements, as MATH 4007, for which additional credit is precluded.

### MATH 5008 [0.5 credit] (MAT 5126)

#### Real Analysis II (Functional Analysis)

Banach and Hilbert spaces, bounded linear operators, dual spaces. Topics selected from: weak-topologies, Alaoglu's theorem, compact operators, differential calculus in Banach spaces, Riesz representation theorems. Prerequisite(s): MATH 5007 (MAT 5125) or permission of the School.

Also offered at the undergraduate level, with different requirements, as MATH 4003, for which additional credit is precluded.

### MATH 5009 [0.5 credit] (MAT 5121)

#### Introduction to Hilbert Space

Geometry of Hilbert Space, spectral theory of linear operators in Hilbert Space.

### MATH 5102 [0.5 credit] (MAT 5148)

#### Group Representations and Applications

An introduction to group representations and character theory, with selected applications.

### MATH 5103 [0.5 credit] (MAT 5146)

#### Rings and Modules

Generalizations of the Wedderburn-Artin theorem and applications, homological algebra.

### MATH 5104 [0.5 credit] (MAT 5143)

#### Lie Algebras

Basic concepts: ideals, homomorphisms, nilpotent, solvable, semi-simple. Representations, universal enveloping algebra. Semi-simple Lie algebras: structure theory, classification, and representation theory.

Prerequisite(s): MATH 5107 (MAT 5141) and MATH 5109 (MAT 5142) or permission of the School.

### MATH 5106 [0.5 credit] (MAT 5145)

#### Group Theory

Fundamental principles as applied to abelian, nilpotent, solvable, free, and finite groups; representations.

Also offered at the undergraduate level, with different requirements, as MATH 4106, for which additional credit is precluded.

**MATH 5107 [0.5 credit] (MAT 5141)****Algebra I: Rings and Modules**

Noetherian and artinian modules and rings. Varieties, Hilbert Basis Theorem, radical ideals, Hilbert Nullstellensatz. Localization and tensor products of modules and algebras. Semisimple rings and modules, Schur's Lemma, Jacobson Density Theorem, Artin-Wedderburn Theorem. Short exact sequences. Free, projective, injective and flat modules.

**MATH 5108 [0.5 credit] (MAT 5147)****Homological Algebra and Category Theory**

Axioms of set theory, categories, functors, natural transformations; free, projective, injective and flat modules; tensor products and homology functors, derived functors; dimension theory.

Also offered at the undergraduate level, with different requirements, as MATH 4108, for which additional credit is precluded.

**MATH 5109 [0.5 credit] (MAT 5142)****Algebra II: Groups and Galois Theory**

Group actions, class equation, Sylow theorems, central, composition and derived series, Jordan-Holder theorem, field extensions and minimal polynomials, algebraic closure, separable extensions, integrality, Galois groups, fundamental theorem of Galois theory, finite fields, cyclotomic field extensions, fundamental theorem of algebra, transcendental extensions.

**MATH 5201 [0.5 credit] (MAT 5150)****Topics in Geometry**

Various axiom systems of geometry. Detailed examinations of at least one modern approach to foundations, with emphasis upon the connections with group theory.

**MATH 5202 [0.5 credit] (MAT 5168)****Homology Theory**

The Eilenberg-Steenrod axioms and their consequences, singular homology theory, applications to topology and algebra.

Prerequisite(s): MATH 5205 (MAT 5151) or permission of the School.

**MATH 5205 [0.5 credit] (MAT 5151)****Topology I**

Topological spaces, product and identification topologies, countability and separation axioms, compactness, connectedness, homotopy, fundamental group, net and filter convergence.

Also offered at the undergraduate level, with different requirements, as MATH 4205, for which additional credit is precluded.

**MATH 5206 [0.5 credit] (MAT 5152)****Topology II**

Covering spaces, homology via the Eilenberg-Steenrod Axioms, applications, construction of a homology functor. Prerequisite(s): MATH 5205 (MAT 5151) or permission of the School.

Also offered at the undergraduate level, with different requirements, as MATH 4206, for which additional credit is precluded.

**MATH 5207 [0.5 credit] (MAT 5169)****Foundations of Geometry**

A study of at least one modern axiom system of Euclidean and non-Euclidean geometry, embedding of hyperbolic and Euclidean geometries in the projective plane, groups of motions, models of non-Euclidean geometry.

**MATH 5208 [0.5 credit] (MAT 5155)****Differentiable Manifolds**

A study of differentiable manifolds from the point of view of either differential topology or differential geometry.

Topics such as smooth mappings, transversality, intersection theory, vector fields on manifolds, Gaussian curvature, Riemannian manifolds, differential forms, tensors, and connections are included.

**MATH 5300 [0.5 credit] (MAT 5160)****Mathematical Cryptography**

Analysis of cryptographic methods used in authentication and data protection, with particular attention to the underlying mathematics, e.g. Algebraic Geometry, Number Theory, and Finite Fields. Advanced topics on Public-Key Cryptography: RSA and integer factorization, Diffie-Hellman, discrete logarithms, elliptic curves. Topics in current research.

**MATH 5301 [0.5 credit] (MAT 5161)****Mathematical Logic**

A basic graduate course in mathematical logic. Propositional and predicate logic, proof theory, Gentzen's Cut-Elimination, completeness, compactness, Henkin models, model theory, arithmetic and undecidability. Special topics (time permitting) depending on interests of instructor and audience.

**MATH 5305 [0.5 credit] (MAT 5163)****Analytic Number Theory**

Dirichlet series, characters, Zeta-functions, prime number theorem, Dirichlet's theorem on primes in arithmetic progressions, binary quadratic forms.

**MATH 5306 [0.5 credit] (MAT 5164)****Algebraic Number Theory**

Algebraic number fields, bases, algebraic integers, integral bases, arithmetic in algebraic number fields, ideal theory, class number.

Also offered at the undergraduate level, with different requirements, as MATH 4306, for which additional credit is precluded.

**MATH 5403 [0.5 credit] (MAT 5187)****Topics in Applied Mathematics****MATH 5405 [0.5 credit] (MAT 5131)****Ordinary Differential Equations**

Linear systems, fundamental solution. Nonlinear systems, existence and uniqueness, flow. Equilibria, periodic solutions, stability. Invariant manifolds and hyperbolic theory. One or two specialized topics taken from, but not limited to: perturbation and asymptotic methods, normal forms and bifurcations, global dynamics.

**MATH 5406 [0.5 credit] (MAT 5133)****Partial Differential Equations**

First-order equations, characteristics method, classification of second-order equations, separation of variables, Green's functions.  $L_p$  and Sobolev spaces, distributions, variational formulation and weak solutions, Lax-Milgram theorem, Galerkin approximation. Parabolic PDEs. Wave equations, hyperbolic systems, nonlinear PDEs, reaction-diffusion equations, infinite-dimensional dynamical systems, regularity.

**MATH 5407 [0.5 credit] (MAT 5134)****Topics in Partial Differential Equations**

Theory of distributions, initial-value problems based on two-dimensional wave equations, Laplace transform, Fourier integral transform, diffusion problems, Helmholtz equation with application to boundary and initial-value problems in cylindrical and spherical coordinates.

Prerequisite(s): MATH 5406 or permission of the School.

Also offered at the undergraduate level, with different requirements, as MATH 4701, for which additional credit is precluded.

**MATH 5408 [0.5 credit] (MAT 5185)****Asymptotic Methods of Applied Mathematics**

Asymptotic series: properties, matching, application to differential equations. Asymptotic expansion of integrals: elementary methods, methods of Laplace, Stationary Phase and Steepest Descent, Watson's Lemma, Riemann-Lebesgue Lemma. Perturbation methods: regular and singular perturbation for differential equations, multiple scale analysis, boundary layer theory, WKB theory.

**MATH 5605 [0.5 credit] (MAT 5165)****Theory of Automata**

Algebraic structure of sequential machines, decomposition of machines; finite automata, formal languages; complexity.

Also offered at the undergraduate level, with different requirements, as MATH 4805/COMP 4805, for which additional credit is precluded.

**MATH 5607 [0.5 credit] (MAT 5324)****Game Theory**

Two-person zero-sum games; infinite games; multi-stage games; differential games; utility theory; two-person general-sum games; bargaining problem; n-person games; games with a continuum of players.

Also offered at the undergraduate level, with different requirements, as MATH 4807, for which additional credit is precluded.

**MATH 5609 [0.5 credit] (MAT 5301)****Topics in Combinatorial Mathematics**

Courses in special topics related to Combinatorial Mathematics, not covered by other graduate courses.

**MATH 5801 [0.5 credit] (MAT 5303)****Linear Optimization**

Linear programming problems; simplex method, upper bounded variables, free variables; duality; postoptimality analysis; linear programs having special structures; integer programming problems; unimodularity; knapsack problem.

**MATH 5803 [0.5 credit] (MAT 5304)****Nonlinear Optimization**

Methods for unconstrained and constrained optimization problems; Kuhn-Tucker conditions; penalty functions; duality; quadratic programming; geometric programming; separable programming; integer nonlinear programming; pseudo-Boolean programming; dynamic programming.

**MATH 5804 [0.5 credit] (MAT 5307)****Topics in Operations Research****MATH 5805 [0.5 credit] (MAT 5308)****Topics in Algorithm Design****MATH 5806 [0.5 credit] (MAT 5180)****Numerical Analysis**

Error analysis for fixed and floating point arithmetic; systems of linear equations; eigen-value problems; sparse matrices; interpolation and approximation, including Fourier approximation; numerical solution of ordinary and partial differential equations.

**MATH 5807 [0.5 credit] (MAT 5167)****Formal Language and Syntax Analysis**

Computability, unsolvable and NP-hard problems. Formal languages, classes of language automata. Principles of compiler design, syntax analysis, parsing (top-down, bottom-up), ambiguity, operator precedence, automatic construction of efficient parsers, LR, LR(O), LR(k), SLR, LL(k). Syntax directed translation.

Prerequisite(s): MATH 5605.

**MATH 5808 [0.5 credit] (MAT 5305)****Combinatorial Optimization I**

Network flow theory and related material. Topics will include shortest paths, minimum spanning trees, maximum flows, minimum cost flows. Optimal matching in bipartite graphs.

**MATH 5809 [0.5 credit] (MAT 5306)****Combinatorial Optimization II**

Topics include optimal matching in non-bipartite graphs, Euler tours, and the Chinese Postman problem. Other extensions of network flows: dynamic flows, multicommodity flows, and flows with gains, bottleneck problems. Matroid optimization. Enumerative and heuristic algorithms for the Traveling Salesman and other problems.

Prerequisite(s): MATH 5808 or permission of the school.

**MATH 5818 [0.5 credit] (MAT 5105)****Discrete Applied Mathematics I: Graph Theory**

Paths and cycles, trees, connectivity, Euler tours and Hamilton cycles, edge colouring, independent sets and cliques, vertex colouring, planar graphs, directed graphs. Selected topics from one or more of the following areas: algebraic graph theory, topological graph theory, random graphs.

**MATH 5819 [0.5 credit] (MAT 5107)****Discrete Applied Mathematics II: Combinatorial Enumeration**

Ordinary and exponential generating functions, product formulas, permutations, rooted trees, cycle index, WZ method. Lagrange inversions, singularity analysis of generating functions and asymptotics. Selected topics from one or more of the following areas: random graphs, random combinatorial structures, hypergeometric functions.

**MATH 5821 [0.5 credit] (MAT 5341)****Quantum Computing**

Space of quantum bits; entanglement. Observables in quantum mechanics. Density matrix and Schmidt decomposition. Quantum cryptography. Classical and quantum logic gates. Quantum Fourier transform. Shor's quantum algorithm for factorization of integers. Also offered at the undergraduate level, with different requirements, as MATH 4821, for which additional credit is precluded.

**MATH 5822 [0.5 credit] (MAT 5343)**  
**Mathematical Aspects of Wavelets and Digital Signal Processing**  
Lossless compression methods. Discrete Fourier transform and Fourier-based compression methods. JPEG and MPEG. Wavelet analysis. Digital filters and discrete wavelet transform. Daubechies wavelets. Wavelet compression.  
Also offered at the undergraduate level, with different requirements, as MATH 4822, for which additional credit is precluded.

**MATH 5900 [0.5 credit] (MAT 5990)**  
**Seminar**

**MATH 5901 [0.5 credit] (MAT 5991)**  
**Directed Studies**

**MATH 5906 [0.5 credit] (MAT 5996)**  
**Research Internship**  
This course affords students the opportunity to undertake research in mathematics as a cooperative project with governmental or industrial sponsors. The grade will be based upon the mathematical content and upon oral and written presentation of results.  
Includes: Experiential Learning Activity  
Prerequisite(s): permission of the graduate director.

**MATH 5909 [2.0 credits] (MAT 7999)**  
**M.Sc. Thesis in Mathematics**  
Includes: Experiential Learning Activity

**MATH 5910 [1.0 credit] (MAT 6997)**  
**M.Sc. Project in Mathematics**  
Project in mathematics supervised by a professor approved by the graduate director resulting in a major report (approximately 30-40 pages), together with a short presentation on the report. Graded by the supervisor and another professor appointed by the graduate director.  
Includes: Experiential Learning Activity  
Precludes additional credit for MATH 5909.

**MATH 5993 [0.0 credit] (MAT 5993)**  
**Research Participation**  
Includes: Experiential Learning Activity

**MATH 6002 [0.5 credit] (MAT 5309)**  
**Harmonic Analysis on Groups**  
Transformation groups; Haar measure; unitary representations of locally compact groups; completeness and compact groups; character theory; decomposition.

**MATH 6008 [0.5 credit] (MAT 5326)**  
**Topics in Analysis**

**MATH 6101 [0.5 credit] (MAT 5327)**  
**Topics in Algebra**

**MATH 6104 [0.5 credit] (MAT 5158)**  
**Lie Groups**  
Matrix groups: one-parameter groups, exponential map, Campbell-Hausdorff formula, Lie algebra of a matrix group, integration on matrix groups. Abstract Lie groups.  
Prerequisite(s): MATH 5007 or permission of the School.

**MATH 6201 [0.5 credit] (MAT 5312)**  
**Topics in Topology**

**MATH 6507 [0.5 credit] (MAT 5319)**  
**Topics in Probability**

**MATH 6806 [0.5 credit] (MAT 5361)**  
**Topics in Mathematical Logic**

**MATH 6807 [0.5 credit] (MAT 5162)**  
**Mathematical Foundations of Computer Science**  
Foundations of functional languages, lambda calculi (typed, polymorphically typed, untyped), Curry-Howard Isomorphism, proofs-as-programs, normalization and rewriting theory, operational semantics, type assignment, introduction to denotational semantics of programs, fixed-point programming.

**MATH 6900 [0.5 credit] (MAT 6990)**  
**Seminar**

**MATH 6901 [0.5 credit] (MAT 6991)**  
**Directed Studies**

**MATH 6909 [0.0 credit] (MAT 9999)**  
**Ph.D. Thesis**  
Includes: Experiential Learning Activity